

Parametrizações.

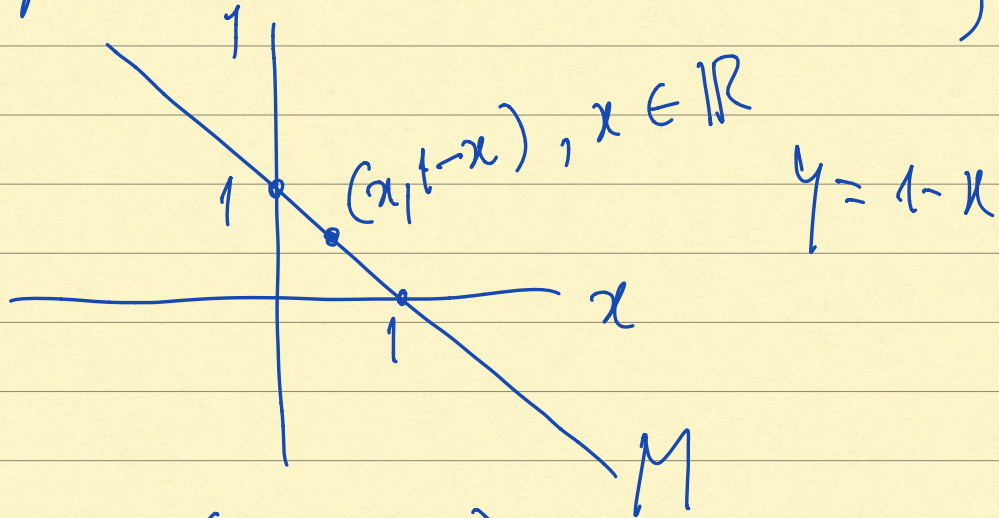
$$g: \mathbb{R}^{n-m} \rightarrow \mathbb{R}^n, C^1, \text{ injetiva}$$

$$\text{car } Dg(\cdot) = n-m$$

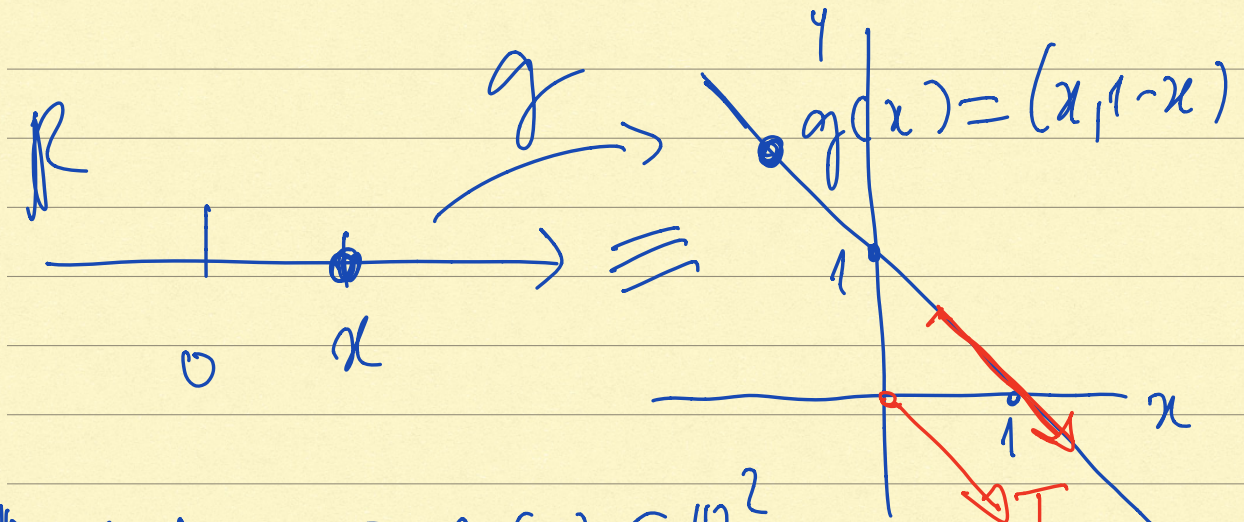
$$M = g(T), \quad T \subset \mathbb{R}^{n-m} \text{ aberto}$$

↳ Variedade $(n-m)$

Exemplo: $M = \left\{ (x, y) \in \mathbb{R}^2 : \underbrace{x+y}_{F(x,y)=1} = 1 \right\}$



$$g(x) = (x, 1-x), \quad x \in \mathbb{R}$$



$$\mathbb{R} \ni x \mapsto g(x) \in \mathbb{R}^2 \quad g(\mathbb{R}) = M$$

$$Dg(x) = g'(x) = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

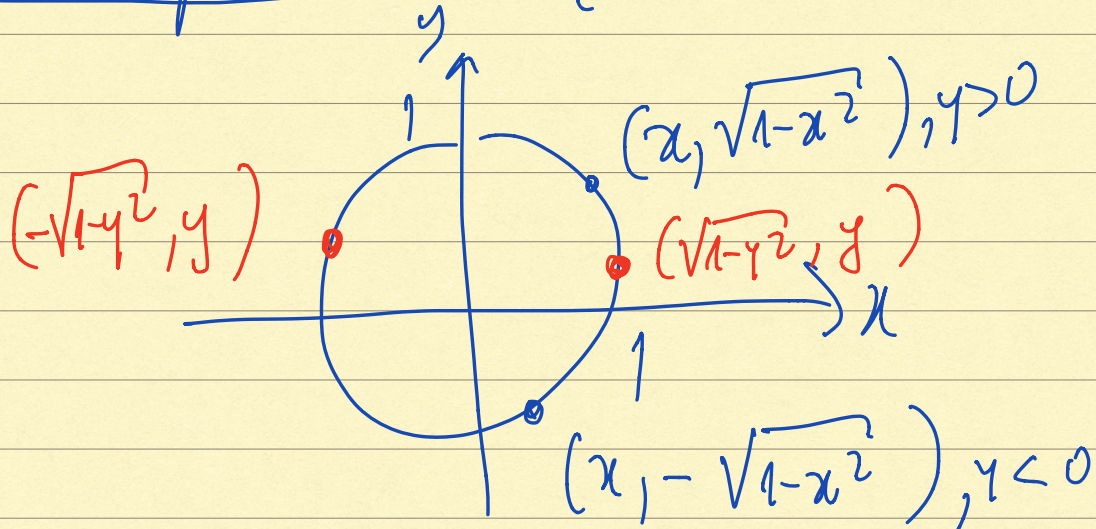
\uparrow
 Tangente

$$F(x, y) = x + y - 1 = 0$$

$$DF(x, y) = \begin{bmatrix} 1 & 1 \end{bmatrix} \quad N, T = 0$$

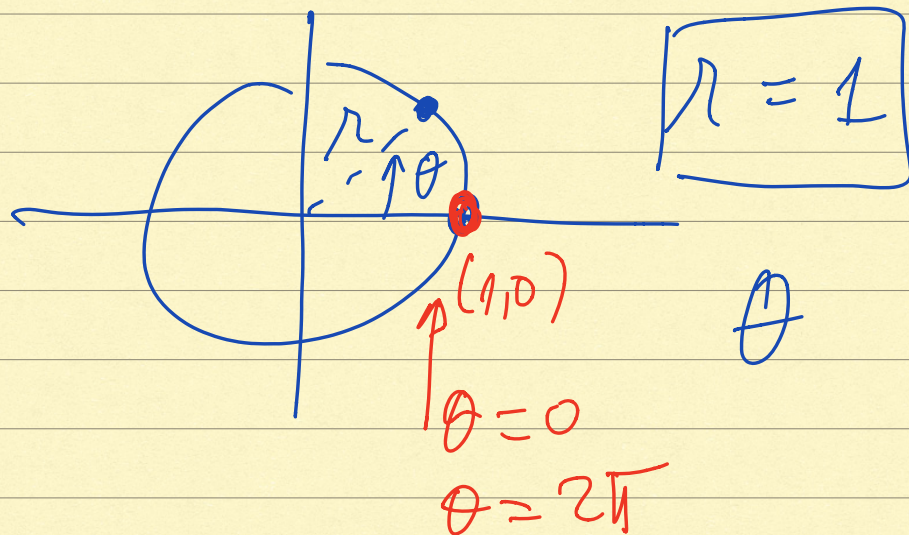
$\underbrace{\hspace{2cm}}_N$

Exemplo 2: $C = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}$



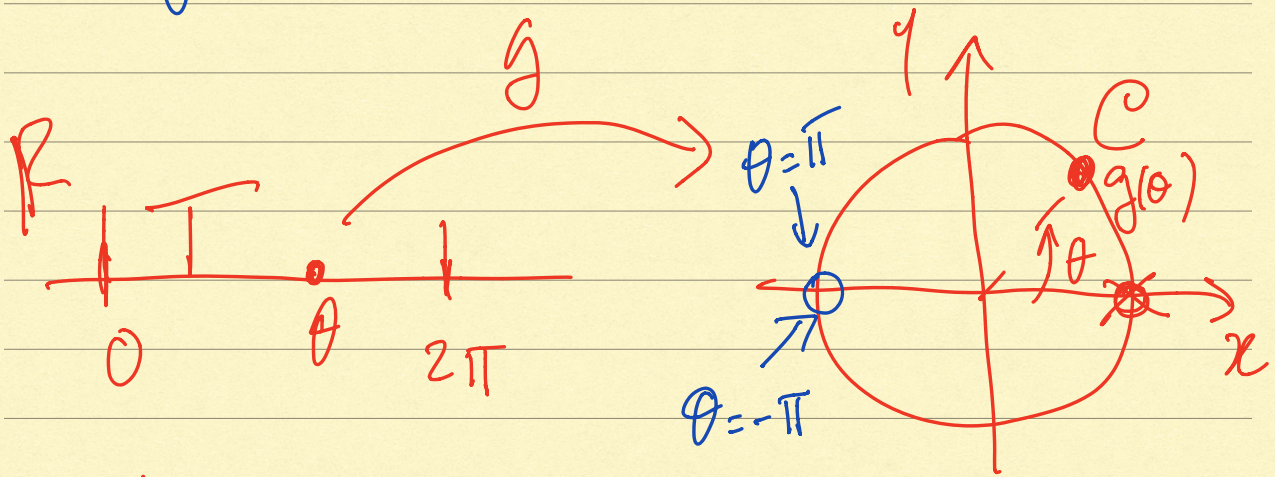
4 pontos \Rightarrow 4 parâmetros

\rightarrow Coordenadas polares (r, θ)



$$\begin{cases} x = r \cos \theta \\ y = r \sin \theta \end{cases} \quad r = 1$$

$$g(\theta) = (\cos \theta, \sin \theta); \quad 0 < \theta < 2\pi$$



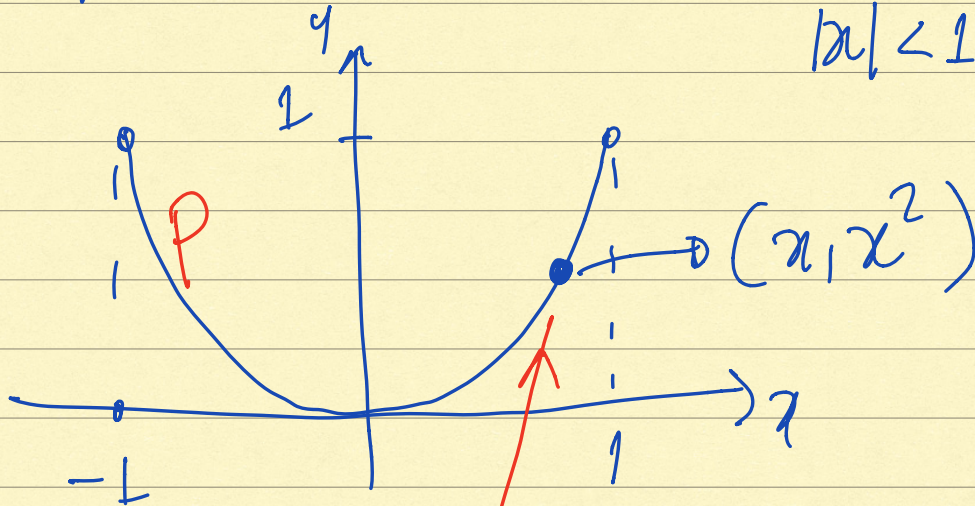
$$C \setminus \{(1, 0)\} \equiv g(T)$$

$$T = \{ \theta \in \mathbb{R} : 0 < \theta < 2\pi \}$$

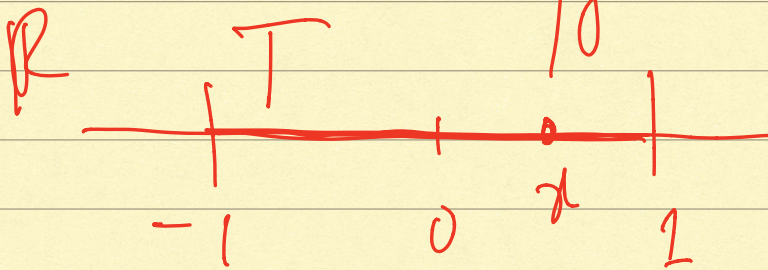
$$h(\theta) = (\cos \theta, \sin \theta), \quad -\pi < \theta < \pi$$

Exemplo 3: $P = \{(x, y) \in \mathbb{R}^2 : y = x^2\}$

$$|x| < 1$$

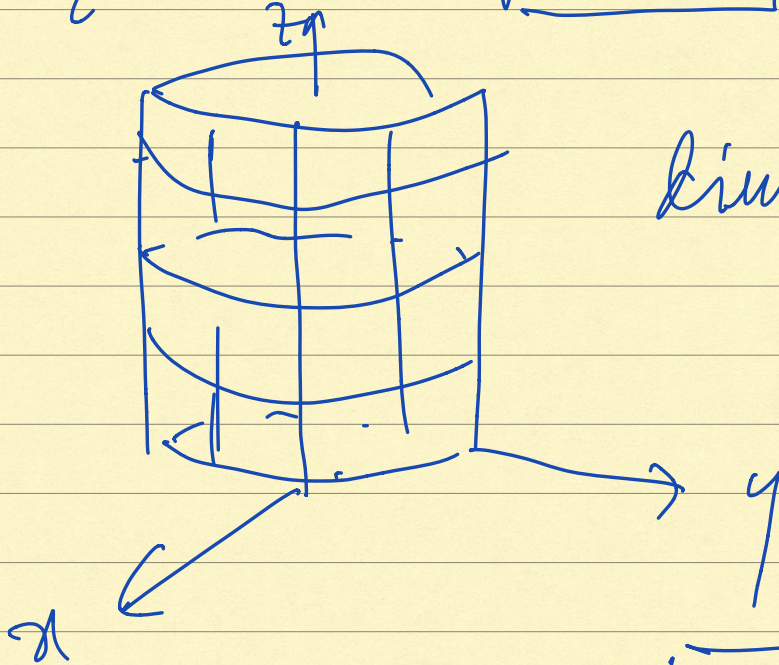


$$g(x) = (x, x^2), \quad -1 < x < 1.$$



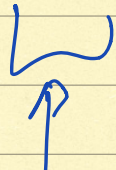
Exemplo 4:

$$M = \{(x, y, z) \in \mathbb{R}^3 : \boxed{x^2 + y^2 = 1}, 0 < z < 1\}$$



$$\dim(M) = 2 //$$

$$(p, \theta, z)$$



parâmetros

$$\rho = \sqrt{x^2 + y^2}$$

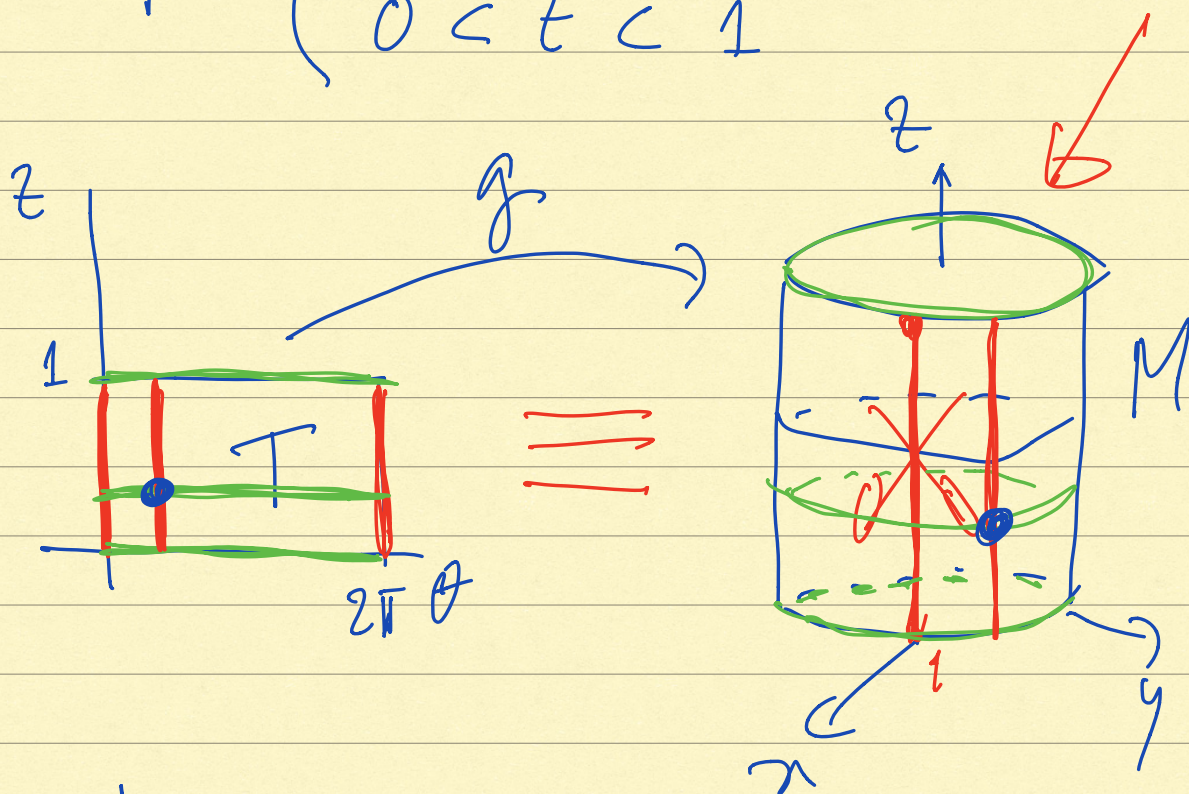
$$\boxed{\rho = 1}$$

$$\left. \begin{array}{l} x = \rho \cos \theta \\ y = \rho \sin \theta \\ z = z \end{array} \right\}$$

$$g(\theta, z) = (x(\theta, z), y(\theta, z), z(\theta, z))$$

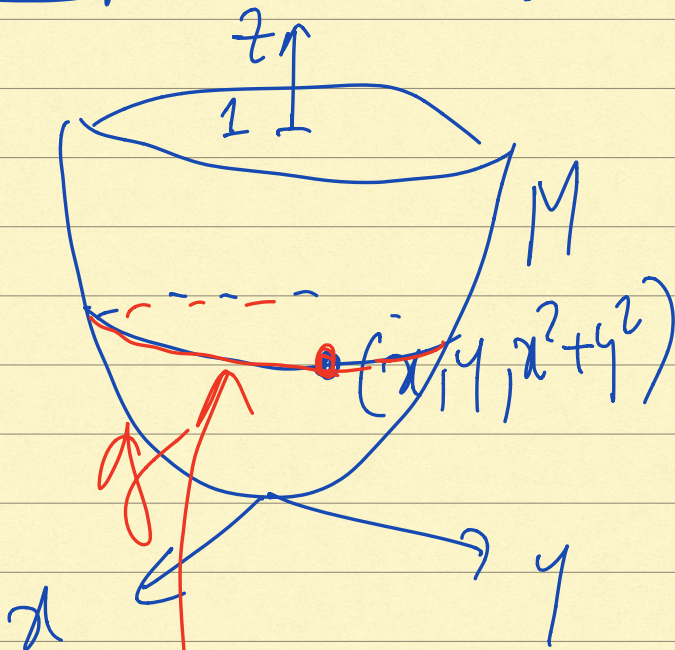
$$= (\cos\theta, \sin\theta, z)$$

$$T: \begin{cases} 0 < \theta < 2\pi \\ 0 < z < 1 \end{cases}$$



$$M \setminus \{(1, 0, z)\} \equiv g(T)$$

Exemplo 5: $M = \{(x, y, z) \in \mathbb{R}^3 : z = x^2 + y^2, x^2 + y^2 < 1\}$

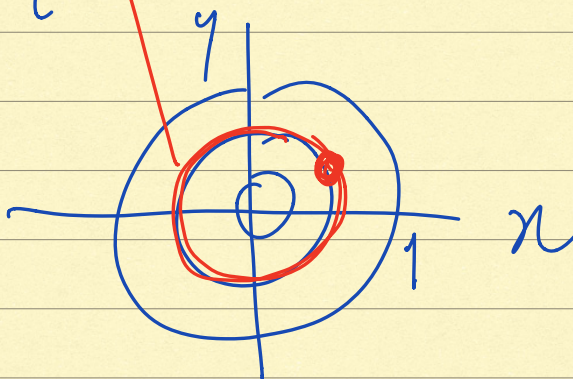


$\dim(M) = 2$

\downarrow
2 parâmetros

$$g(x, y) = (x, y, x^2 + y^2)$$

$$T = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 < 1\}$$



$$M = g(T)$$

$$(\rho, \theta, z)$$

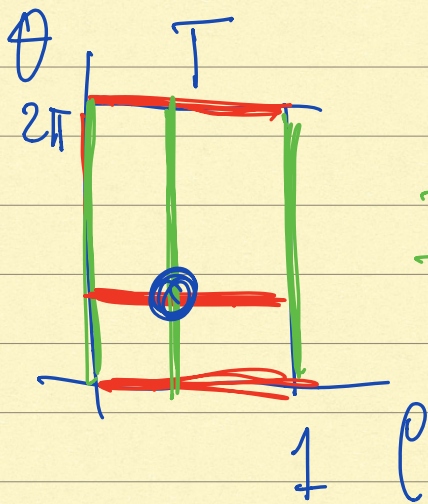
$$z = x^2 + y^2; x^2 + y^2 < 1$$

$$z = \rho^2; \rho < 1$$

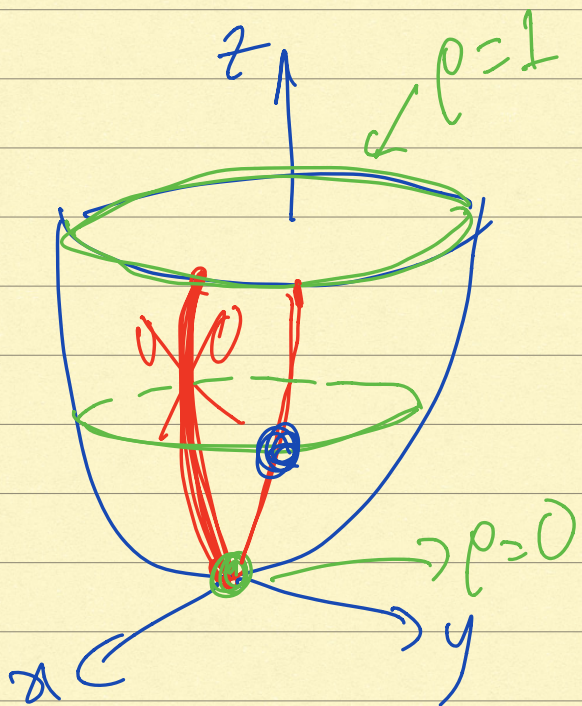
$$g(\rho, \theta) = (x(\rho, \theta), y(\rho, \theta), z(\rho, \theta))$$

$$= (\rho \cos \theta, \rho \sin \theta, \rho^2)$$

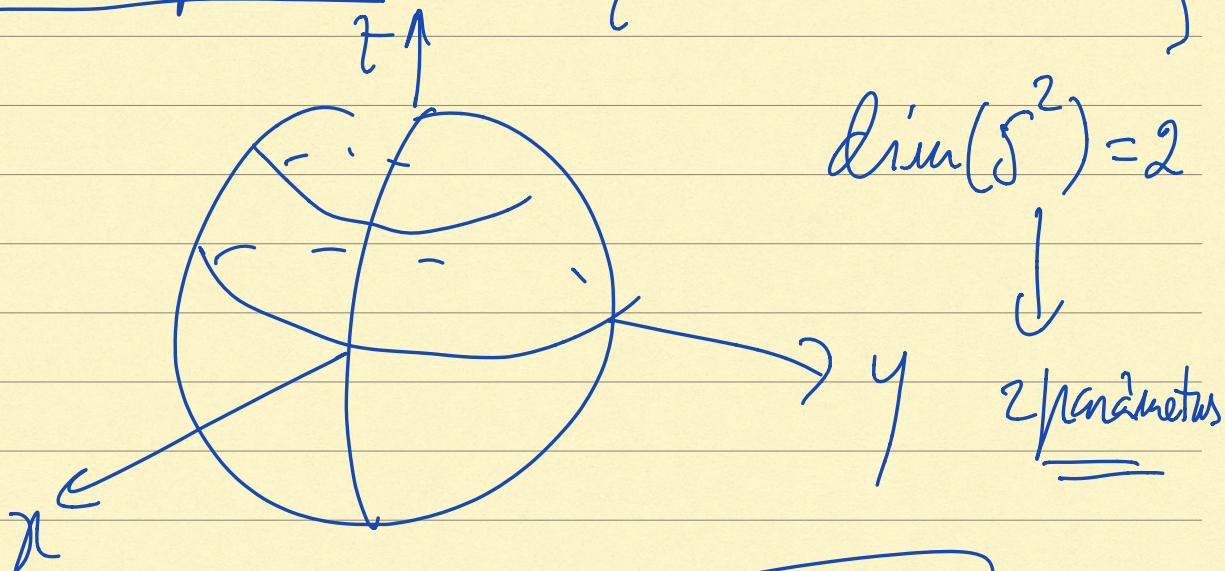
$$\begin{cases} 0 < \theta < 2\pi \\ 0 < \rho < 1 \end{cases}$$



g



Exemplo 6: $S^2 = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 1\}$



(r, θ, φ)

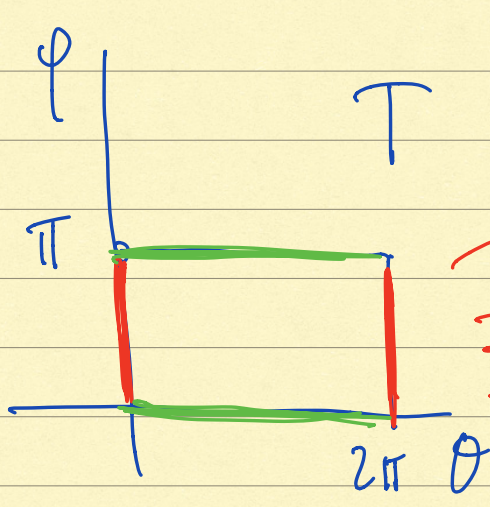
$$r = 1$$

parâmetros.

$$g(\theta, \varphi) = (x(\theta, \varphi), y(\theta, \varphi), z(\theta, \varphi))$$

$$= (\sin \varphi \cos \theta, \sin \varphi \sin \theta, \cos \varphi)$$

$$T: \begin{cases} 0 < \theta < 2\pi \\ 0 < \varphi < \pi \end{cases}$$



g
 III

